The critical exponents $\nu$ , $\nu_{||}$ and $\nu_{\text{perpendicular to}}$ of directed Levy flight: a Monte Carlo simulation study



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# The critical exponents $\nu$, $\nu_{\parallel}$ and $\nu_{\perp}$ of directed Levy flight: a Monte Carlo simulation study†

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**Abstract.** The critical behaviour of fully directed Levy flight on a square lattice is studied using the Monte Carlo method. The obtained critical exponents $\nu$, $\nu_{\parallel}$ and $\nu_{\perp}$ are independent of the parameter $u$. This seems to be interesting compared with the Levy flight previously discussed by Halley and Nakanishi, for which $\nu$ depends on $u$ in addition to $d$. It also indicates that the introduction of the direction plays a dominant role in directed Levy flight just as in directed SAW, with $\nu$, $\nu_{\parallel}$ and $\nu_{\perp}$ independent of $d$.

Critical phenomena in systems with power-dependent long-range forces have been studied since the early days of renormalisation group theory [1, 2]. By the $\varepsilon$ expansions, for fixed $d$ these authors obtained

$$\nu = (2/d)[(1-\varepsilon/2d+(\varepsilon^2/d^2)(304-5d^2)/256+\ldots] \tag{1}$$

which corresponds to the following expansion for $0<u<2$

$$\nu = \begin{cases} (1/u)[1+\varepsilon/4d+(\varepsilon^2/64u^2)(19-\frac{5}{4}u^2+\ldots)] & d<2u \\ 1/u & d>2u. \end{cases} \tag{2}$$

Only recently, however, have the simplest such systems, namely node-avoiding Levy flights, been simulated by the Monte Carlo technique in order to verify these predictions [3].

Levy flight is similar to random walk, except that the steps are not necessarily to next neighbours [4]. Instead, the probability for a step to have a step length $r$ is proportional to $1/r^{1+u}$ with $0<u<2$, for $r\to\infty$. The properties of such a random walk are strikingly different from those of ordinary random walks. When $0<u<1$, the mean and the mean-square displacement per step cannot be well defined since $\int \boldsymbol{r}P(\boldsymbol{r})\,\mathrm{d}^s\boldsymbol{r}$ and $\int \boldsymbol{r}^2P(\boldsymbol{r})\,\mathrm{d}^s\boldsymbol{r}$ are not absolutely convergent. In the limit of a large number of steps the Hausdorff-Besicovitch dimension of the 'trail' consisting of the endpoints of the steps is $u$. In analogy to the relation between self-avoiding random walks and spin systems found by de Gennes [5], the critical behaviour of node-avoiding Levy

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flight is described by the $n$-vector model with $n=0$. This model can be written as

$$H=\sum_{i,j}^{x} J_{ij}\boldsymbol{S}_i\cdot\boldsymbol{S}_j$$
$$n=\sum_{\alpha=1}^{n}(S_i^\alpha)^2 \qquad n\to 0. \tag{3}$$

Here $H$ is the Hamiltonian, $J_{ij}$ is zero unless $i$ and $j$ lie along one of the $d$ orthogonal directions in the two-dimensional lattice and is $(J/\beta)r^{-(1+u)}$, where $r$ is the distance between sites $i$ and $j$ and $\beta$ is the reciprocal of the temperature. Similar to the derivation of the equivalence of the nearest-neighbour $n$-vector model to a self-avoiding walk in the limit $n\to 0$, one has for a node-avoiding Levy flight that

$$\langle S_i^\alpha S_j^\alpha\rangle=\sum_N (J/\beta)^N\bar{\eta}_N(r_{ij}) \tag{4}$$

where

$$\bar{\eta}_N=\sum_{\{r_{lm}\}}\prod_{lm} Plm\eta_N(r_{ij}\{r_{lm}\}) \tag{5}$$

with $Plm=r_{lm}^{1+u}$, $\eta_N(r_{ij},\{r_{lm}\})$ is the number of node-avoiding Levy flights between $\boldsymbol{r}_i$ and $\boldsymbol{r}_j$ with $N$ steps of lengths given by $\{\boldsymbol{r}_{lm}\}$.

In this paper, we study an extension of node-avoiding Levy flight to include a preferred direction in the path of the walk, which we call 'directed Levy flight'. The meaning of this direction constraint is illustrated in figure 1 where a disallowed step on a square lattice is shown. In recent years systems with power-dependent short-range forces and directionally dependent critical phenomena have been the focus of much attention. These systems are directed percolation [6], directed lattice animals [7] and directed self-avoiding walks [8]. The introduction of preferred direction in such systems gives rise to two independent correlation lengths $R_\parallel$ and $R_\perp$, parallel and perpendicular to the preferred direction respectively. In the case of directed SAW the corresponding exponents $\nu_\parallel$ and $\nu_\perp$ have been obtained exactly with the values $\nu_\parallel=1.0$ and $\nu_\perp=0.5$ for all dimensions $d\geqslant 2$ [9]. These results indicate that the directed self-avoiding random walk consists of a one-dimensional directed self-avoiding random walk and an ordinary random walk. What behaviour will the directed Levy flight show?

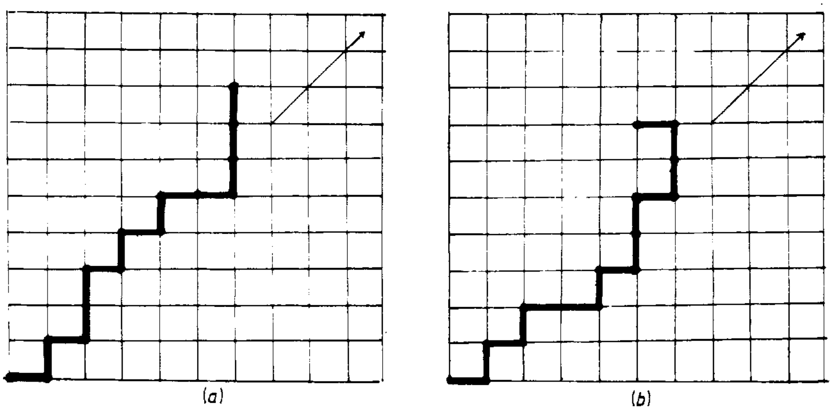

**Figure 1.** Illustration of the constraint which defines a fully directed Levy flight, including paths of the type illustrated in ($a$) but not those shown in ($b$).

In order to test the effect of direction on directed Levy flight we have performed a Monte Carlo simulation in two dimensions. The probabilities of making steps with the step length $r$ were chosen as $[r^{-u}-(r+1)^{-u}]$. This is because the probability for a step to have a length greater than some $r$ is assumed to decrease as $r$. The maximum step length was chosen as six. The total number of directed Levy flights for $N=20$, 40, 60, 80, 100 are $20N^2$ for each value of $u$.

We calculated the correlation lengths $\overline{R^2}$, $\overline{R_\parallel^2}$ and $\overline{R_\perp^2}$ (see figure 2) for $N$ steps on a Honeywell machine. Expressing $\overline{R_\parallel^2}$, $\overline{R_\perp^2}$ and $\overline{R^2}$ for $N\to\infty$ as in [10]

$$\overline{R_\parallel^2}\sim N^{2\nu_\parallel} \qquad \overline{R_\perp^2}\sim N^{2\nu_\perp} \qquad \overline{R^2}\sim N^{2\nu} \tag{6}$$

we find for a directed Levy flight in two dimensions (see figure 3) that

| | | | |
|---|---|---|---|
| $u=0.5$ | $\nu=0.975\pm0.002$ | $\nu_\perp=0.504\pm0.002$ | $\nu_\parallel=0.992\pm0.003$ |
| $u=1.2$ | $\nu=0.975\pm0.003$ | $\nu_\perp=0.502\pm0.002$ | $\nu_\parallel=0.993\pm0.002$ |
| $u=1.4$ | $\nu=0.976\pm0.004$ | $\nu_\perp=0.502\pm0.001$ | $\nu_\parallel=0.992\pm0.002$ |
| $u=1.6$ | $\nu=0.975\pm0.002$ | $\nu_\perp=0.502\pm0.002$ | $\nu_\parallel=0.991\pm0.003$ |
| $u=2.5$ | $\nu=0.980\pm0.003$ | $\nu_\perp=0.500\pm0.001$ | $\nu_\parallel=0.994\pm0.002$ |

compared with $\nu=0.97$, $\nu_\perp=0.50$ and $\nu_\parallel=1.00$ for directed SAW [11] obtained with the same finite step number $N$ as for the directed Levy flight above. For $N\to\infty$, we believe that both cases will approach the results $\nu=\nu_\parallel=1.0$ and $\nu_\perp=0.5$ [12].

These results indicate explicitly that directed Levy flight belongs to the same universality class as directed SAW and that the critical exponents $\nu$, $\nu_\parallel$ and $\nu_\perp$ are independent of the parameter $u$. Such a result seems to be interesting compared with the Levy flight discussed by Halley and Nakanishi for which $\nu$ depends on both $u$ and $d$. We know [11, 12] that fully directed SAW and partially directed SAW belong to the same universality class and that two-dimensional directed SAW belongs to the same universality class as three-dimensional directed SAW, which means that the introduction of the 'direction' plays a dominant role in the directed SAW. Our results indicate that the 'direction' still plays the dominant role in the two-dimensional directed Levy flight. Whether or not the three-dimensional directed Levy flight has the same behaviour will be reported in another paper.

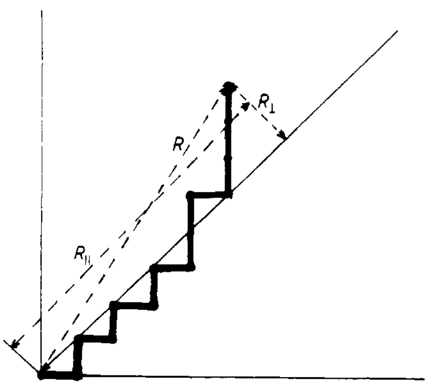


**Figure 2.** The correlation lengths $R$, $R_\parallel$ and $R_\perp$ for a fully directed Levy flight.

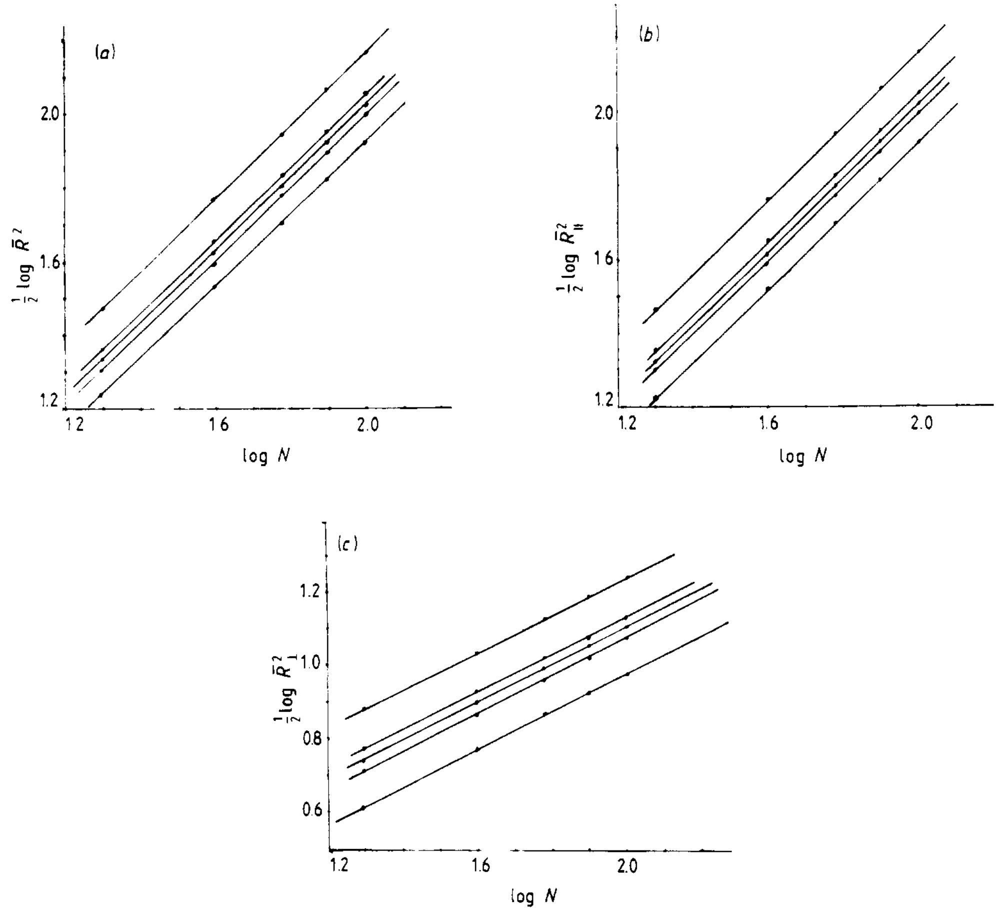


**Figure 3.** Plots of ($a$) $\log \overline{R^2}$, ($b$) $\log \overline{R^2_\parallel}$ and ($c$) $\log \overline{R^2_\perp}$ against $\log N$ for, from top to bottom, $u = 0.5$, 1.2, 1.4, 1.6 and 2.5. The slopes of the straight-line fits give the exponents for a fully directed Levy flight in two dimensions.

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